Exercises 3.1

[07-04-24-MT-11]

Find the solution sets for the following equations.

1. $3 \sin x + 2 = 0.$
2. $4 \cos y - 3 = 0.$
3. $2 + \tan z = 0.$
4. $4 \sin \theta - 1 = 0.$
5. $5 \cos \alpha + 1 = 0.$
6. $3 \tan x - 2 = 0.$
7. $2 \sin y - 5 = 0.$
8. $3 \cos z - 1 = 0.$
9. $3 \tan \theta - 7 = 0.$
10. $2 \sec \alpha - 3 = 0.$
11. $3 \csc x - 1 = 0.$
12. $2 \cot y + 5 = 0.$
13. $3 - \sin z = -2 + 5 \sin z.$
14. $2 + \cos \theta = 1 + 3 \cos \theta.$
15. $\tan \alpha + 2 = 3 - \tan \alpha.$
16. $\sin x - 3 = 3 \sin x - 2.$
17. $\cos y - 2 = 4 \cos y + 3.$
18. $2 + 3 \tan z = 4 - 2 \tan z.$
19. $\sec \theta + 2 = 2 \sec \theta + 1.$
20. $\csc \alpha - 3 = 4 + 2 \csc \alpha.$
21. $2 + 3 \cot \beta = 6 - 2 \cot \beta.$
22. $\sin^2 x - \sin x - 2 = 0.$
23. $3 \cos^2 y - 4 \cos y + 1 = 0.$
24. $2 \tan^2 z - 5 \tan z + 3 = 0.$
25. $6 \sin^2 \alpha + 5 \sin \alpha - 1 = 0.$
26. $12 \cos^2 \theta - 7 \cos \theta + 1 = 0.$
27. $\tan^2 \beta - 9 = 0.$
28. $\sec^2 \gamma + 4 \sec \gamma + 4 = 0.$
29. $2 \csc^2 t - 3 = 0.$
30. $\cot^2 w - 4 \cot w + 3 = 0.$

Answers

Exercises 3.1

1. $S = \{x \mid x = -.73 + 2n\pi, n \in J\} \cup \{x \mid x = \pi + .73 + 2n\pi, n \in J\}.$
3. $S = \{z \mid z = -1.107 + n\pi, n \in J\}.$
5. $S = \{\alpha \mid \alpha = \pi - 1.369 + 2n\pi, n \in J\} \cup \{\alpha \mid \alpha = \pi + 1.369 + 2n\pi, n \in J\}.$
7. $S = \emptyset$, $\frac{5}{2}$ is not in the range of the sine function.
9. $S = \{\theta \mid \theta = 1.166 + n\pi, n \in J\}.$
11. $S = \emptyset.$
13. $S = \{z \mid z = .985 + 2n\pi, n \in J\} \cup \{z \mid z = \pi - .985 + 2n\pi, n \in J\}.$
15. $S = \{\alpha \mid \alpha = .464 + n\pi, n \in J\}.$
17. $S = \emptyset.$
19. $S = \{\theta \mid \theta = 2n\pi, n \in J\}.$
21. $S = \{\beta \mid \beta = .896 + n\pi, n \in J\}.$
23. $S = \{y \mid y = 2n\pi, n \in J\} \cup \{y \mid y = 1.231 + 2n\pi, n \in J\} \cup \{y \mid y = -1.231 + 2n\pi, n \in J\}.$
25. $S = \left\{\alpha \mid \alpha = \frac{3\pi}{2} + 2n\pi, n \in J\right\} \cup \{\alpha \mid \alpha = .167 + 2n\pi, n \in J\} \cup \{\alpha \mid \alpha = \pi - .167 + 2n\pi, n \in J\}.$
27. $S = \{\beta \mid \beta = 1.249 + n\pi, n \in J\} \cup \{\beta \mid \beta = -1.249 + n\pi, n \in J\}.$
29. $S = \{t \mid t = .955 + 2n\pi, n \in J\} \cup \{t \mid t = \pi - .955 + 2n\pi, n \in J\} \cup \{t \mid t = \pi + .955 + 2n\pi, n \in J\} \cup \{t \mid t = -.955 + 2n\pi, n \in J\}.$

Exercises 3.2

Find the solution sets for the following equations.

1. $2 \sin 3x - 1 = \sin 3x - 2.$
2. $2 \cos \frac{\alpha}{2} + 1 = 4 \cos \frac{\alpha}{2} + 2.$
3. $3 \tan 2\beta - 1 = 4 \tan 2\beta - 2.$
4. $4 \sin \frac{y}{3} + 2 = 2 - \sin \frac{y}{3}.$
5. $3 \cos 3z - 1 = 2 \cos 3z + 3.$
6. $6 \tan \frac{x}{5} - 1 = \tan \frac{x}{5} - 3.$
7. $2 \sec 4y + 3 = \sec 4y + 5.$
8. $3 \csc \frac{\alpha}{2} + 1 = 1 - 4 \csc \frac{\alpha}{2}.$
9. $5 \cot 5\beta - 1 = 3 + \cot 5\beta.$
10. $4 \sin 2z - 1 = \sin 2z + 3.$
11. $\cos 2x - 1 = 3 \cos 2x - 1.$
12. $3 \tan \frac{\beta}{3} = \sqrt{3}.$
13. $2 \csc 2\alpha - 1 = 3 \csc 2\alpha + 4.$
14. $3 \csc \frac{y}{3} - 3 = 3 - \csc \frac{y}{3}.$
15. $\cot \frac{z}{4} - 4 = 4 \cot \frac{z}{4} - 1.$
16. $\cos x - \sin x = 1.$
17. $\tan y - \sec y = 1.$
18. $\csc \alpha - \cot \alpha = 1.$
19. $\sin^2 \beta - \cos \beta + 1 = 0.$
20. $\tan^2 z + \sec z - 3 = 0.$
21. $\cot^2 x + 4 = 2 \csc^2 x.$
22. $\sin y - 3 \cos y = 0.$
23. $\sec \alpha - \tan \alpha = 0.$
24. $\csc \beta - 2 \cot \beta = 0.$
25. $\sin z + 2 \cos z = 2.$
26. $\sec x - 3 \tan x = -1.$
27. $2 \csc y - 2 \cot y = 3.$
28. $\cos^2 \alpha + \sin \alpha = -1.$
29. $\sec^2 \beta - \tan \beta = 3.$
30. $\csc^2 z + \cot z = 3.$
31. $\sin \frac{\alpha}{2} - \csc \frac{\alpha}{2} = 0.$
32. $\cos 2\beta + \sec 2\beta = -2.$
33. $\tan 3x + 2 \cot 3x = 3.$

Answers

Exercises 3.2

1. $S = \left\{x \mid x = \frac{\pi}{2} + \frac{2n\pi}{3}, n \in J\right\}.$
3. $S = \left\{\beta \mid \beta = \frac{\pi}{8} + \frac{n\pi}{2}, n \in J\right\}.$
5. $S = \emptyset.$
7. $S = \left\{y \mid y = \frac{\pi}{12} + \frac{n\pi}{2}, n \in J\right\} \cup \left\{y \mid y = -\frac{\pi}{12} + \frac{n\pi}{2}, n \in J\right\}.$
9. $S = \left\{\beta \mid \beta = \frac{\pi}{20} + \frac{n\pi}{5}, n \in J\right\}.$
11. $S = \left\{x \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in J\right\}.$
13. $S = \{\alpha \mid \alpha = -.101 + n\pi, n \in J\} \cup \left\{\alpha \mid \alpha = \frac{\pi}{2} + .101 + n\pi, n \in J\right\}.$
15. $S = \{z \mid z = 3\pi + 4n\pi, n \in J\}.$
17. $S = \{y \mid y = \pi + 2n\pi, n \in J\}.$
19. $S = \{\beta \mid \beta = 2n\pi, n \in J\}.$
21. $S = \{x \mid x = .616 + n\pi, n \in J\} \cup \{x \mid x = -.616 + n\pi, n \in J\}.$
23. $S = \emptyset.$
25. $S = \{z \mid z = 2n\pi, n \in J\} \cup \{z \mid z = .927 + 2n\pi, n \in J\}.$
27. $S = \{y \mid y = \pi - 1.176 + 2n\pi, n \in J\}.$
29. $S = \left\{\beta \mid \beta = \frac{3\pi}{4} + n\pi, n \in J\right\} \cup \{\beta \mid \beta = 1.107 + n\pi, n \in J\}.$
31. $S = \{\alpha \mid \alpha = \pi + 2n\pi, n \in J\}.$
33. $S = \left\{x \mid x = \frac{\pi}{12} + \frac{n\pi}{3}, n \in J\right\} \cup \left\{x \mid x = .369 + \frac{n\pi}{3}, n \in J\right\}.$

Exercises 3.3

Find the solution sets for the following equations.

1. $\sin 4\theta = -\sin 3\theta$.
2. $\cos 3y = \cos 6y$.
3. $\tan 3x = -\tan x$.
4. $\sin 5\alpha = \sin \alpha$.
5. $\cos 4\beta = -\cos 3\beta$.
6. $\tan 2z = \tan 5z$.
7. $\sec 2y = \sec y$.
8. $\csc 3\theta = -\csc 2\theta$.
9. $\cot 5\alpha = \cot 2\alpha$.
10. $\cos 2x = \sin 5x$.
11. $\sin 4z = -\cos 3z$.
12. $\cos 6\beta = \sin 3\beta$.
13. $\tan 2x = \cot 3x$.
14. $\sec 5\alpha = \csc 3\alpha$.
15. $\csc 2\theta = -\sec 4\theta$.

Answers

Exercises 3.3

1. $S = \left\{\theta \mid \theta = \frac{2n\pi}{7}, n \in J\right\} \cup \{\theta \mid \theta = \pi + 2n\pi, n \in J\}$.
3. $S = \left\{x \mid x = \frac{n\pi}{4}, n \in J, n \neq 4k + 2, k \in J\right\}$.
5. $S = \left\{\beta \mid \beta = \frac{\pi}{7} + \frac{2n\pi}{7}, n \in J\right\} \cup \{\beta \mid \beta = \pi + 2n\pi, n \in J\}$.
7. $S = \left\{y \mid y = \frac{2n\pi}{3}, n \in J\right\}$.
9. $S = \left\{\alpha \mid \alpha = \frac{n\pi}{3}, n \in J, n \neq 3k, k \in J\right\}$.
11. $S = \left\{z \mid z = -\frac{\pi}{2} + 2n\pi, n \in J\right\} \cup \left\{z \mid z = \frac{3\pi}{14} + \frac{2n\pi}{7}, n \in J\right\}$.
13. $S = \left\{x \mid x = \frac{\pi}{5} + \frac{2n\pi}{5}, n \in J\right\}$. Mistake
15. $S = \left\{\theta \mid \theta = \frac{\pi}{4} + \frac{n\pi}{3}, n \in J\right\}$.

Review Exercises

Find the solution set for each of the following equations.

1. $5 \sin x + 2 = 0$.
2. $4 \sin 3\beta = 2$.
3. $3 \sin^2 \alpha - 5 \sin \alpha + 2 = 0$.
4. $6 \cos z + 1 = 0$.
5. $\sin 2x - \sin 3x = 0$.
6. $\sin x + \cos x = 1$.
7. $3 \cos 2\beta + 1 = 0$.
8. $4 \cos^2 x - 1 = 0$.
9. $3 - 2 \tan \alpha = -1$.
10. $\cos 4\theta + \cos 5\theta = 0$.
11. $\cot y - \csc y = 1$.
12. $4 \sin\left(\alpha + \frac{\pi}{4}\right) = -2$.
13. $3 \tan 3\gamma - \sqrt{3} = 0$.
14. $3 + 5 \cot z = 2 \cot z - 5$.
15. $\tan^2 x - 7 \tan x + 12 = 0$.
16. $\sin 4\theta + \sin \theta = 0$.
17. $\sin 6\theta - 4 = 2 \sin 6\theta - 3$.
18. $\cos\left(\gamma - \frac{\pi}{2}\right) = 1$.
19. $4 \sec z - 2 = 5 \sec z$.
20. $2 \sin^2 \alpha - 3 \sin \alpha + 1 = 0$.
21. $\sec x + \tan x = 1$.
22. $3 - 2 \csc x = 6 - \csc x$.
23. $\cos 2\theta - \cos 7\theta = 0$.
24. $4 \cos 2\beta - 1 = \cos 2\beta + 2$.
25. $5 \sin\left(2x + \frac{2\pi}{3}\right) - 4 = 0$.

Answers

Review Exercises

1. $S = \{x \mid x = \pi + .412 + 2n\pi, n \in J\} \cup \{x \mid x = -.412 + 2n\pi, n \in J\}$.
3. $S = \left\{\alpha \mid \alpha = \frac{\pi}{2} + 2n\pi, n \in J\right\} \cup \{\alpha \mid \alpha = .730 + 2n\pi, n \in J\} \cup$
   $\{\alpha \mid \alpha = \pi - .730 + 2n\pi, n \in J\}$.
5. $S = \{x \mid x = 2n\pi, n \in J\} \cup \left\{x \mid x = \frac{\pi}{5} + \frac{2n\pi}{5}, n \in J\right\}$.
7. $S = \left\{\beta \mid \beta = \frac{\pi}{2} + .616 + n\pi, n \in J\right\} \cup \left\{\beta \mid \beta = \frac{\pi}{2} - .616 + n\pi, n \in J\right\}$.
9. $S = \{\alpha \mid \alpha = 1.107 + n\pi, n \in J\}$.
11. $S = \left\{y \mid y = -\frac{\pi}{2} + 2n\pi, n \in J\right\}$.
13. $S = \left\{\gamma \mid \gamma = \frac{\pi}{18} + \frac{n\pi}{3}, n \in J\right\}$.
15. $S = \{x \mid x = 1.326 + n\pi, n \in J\} \cup \{x \mid x = 1.249 + n\pi, n \in J\}$.
17. $S = \left\{\theta \mid \theta = -\frac{\pi}{12} + \frac{n\pi}{3}, n \in J\right\}$.
19. $S = \left\{z \mid z = \frac{2\pi}{3} + 2n\pi, n \in J\right\} \cup \left\{z \mid z = \frac{4\pi}{3} + 2n\pi, n \in J\right\}$.
21. $S = \{x \mid x = 2n\pi, n \in J\}$.
23. $S = \left\{\theta \mid \theta = \frac{2n\pi}{5}, n \in J\right\} \cup \left\{\theta \mid \theta = \frac{2n\pi}{9}, n \in J\right\}$.
25. $S = \left\{x \mid x = .464 - \frac{\pi}{3} - n\pi, n \in J\right\} \cup \left\{x \mid x = \frac{\pi}{6} - .464 + n\pi, n \in J\right\}$.